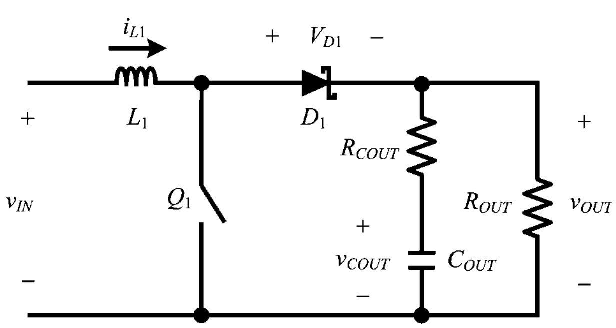
Modeling and Design of Current Mode Control Boost Converters

National Semiconductor
Application Note 1994
L.K.Wong and T.K.Man
March 3, 2010

Introduction

This application note presents a detail modeling and design of current mode control boost converters operating in the continuous conduction mode (CCM). Based on the derived small signal models, the design of a lag compensator for current mode control boost converters will be detailed. The LM3478 boost controller will be used in the example. Simulation and hardware measurement of frequency responses will be shown.

Basic Operation of a Boost Converter



30103401

FIGURE 1. An Open Loop Boost Converter

Figure 1 shows an open loop boost converter with an inductor L_1, a diode D_1, an output capacitor C_{OUT} with an equivalent series resistance R_{COUT}. It is assumed that the load is a resistor R_{OUT}, and the switch Q_1 is ideal. Let v_{IN}, v_{OUT}, and v_{COUT} be the input voltage, output voltage, and the voltage across C_{OUT}, i_{L1} be the current through L_1, and V_{D1} be the forward voltage drop of D_1 when D_1 is turned on. Under the CCM, when Q_1 is turned on, the state equations are

$$v_{IN} = L_1 \frac{d}{dt} i_{L1}, \tag{1}$$

$$\frac{v_{OUT}}{R_{OUT}} = -C_{OUT}\frac{d}{dt}v_{COUT} \tag{2}$$

Also, the output equation is

$$v_{OUT} = v_{COUT} + C_{OUT}\frac{d}{dt}v_{COUT}R_{COUT} \tag{3}$$

Similarly, when Q_1 is turned off, the output equation of (3) still holds, while the state equations become

$$v_{IN} = L_1\frac{d}{dt}i_{L1} + V_{D1} + v_{OUT}, \tag{4}$$

$$\frac{v_{OUT}}{R_{OUT}} = i_{L1} - C_{OUT}\frac{d}{dt}v_{COUT} \tag{5}$$

Modeling of an Open Loop Boost Converter

To obtain a small signal model of boost converters, it is required to apply the averaging technique, perturbation, and the linearization technique. First, by applying the averaging technique, the averaged state equations and output equation are

$$\bar{v}_{IN} = L_1\frac{d}{dt}\bar{i}_{L1} + (1-\bar{d})V_{D1} + (1-\bar{d})\bar{v}_{OUT}, \tag{6}$$

$$\frac{\bar{v}_{OUT}}{R_{OUT}} = (1-\bar{d})\bar{i}_{L1} - C_{OUT}\frac{d}{dt}\bar{v}_{COUT}, \tag{7}$$

$$\bar{v}_{OUT} = \bar{v}_{COUT} + R_{COUT}C_{OUT}\frac{d}{dt}\bar{v}_{COUT} \tag{8}$$

where $\bar{d}$ is the duty cycle, $\bar{x}$ is the averaged variable for the variable x (which can represent v_{IN}, v_{OUT}, v_{COUT}, and i_{L1}). Second, by applying small signal perturbations to (6) to (8), i.e. let

$$\bar{x} = \bar{X} + \tilde{x},$$

where $\bar{X}$ and $\tilde{x}$ are nominal (DC) and perturbed (AC) variables,

$$\bar{V}_{IN} + \tilde{v}_{IN} = L_1\frac{d}{dt}(\bar{I}_{L1} + \tilde{i}_{L1}) + (1-\bar{D}-\tilde{d})V_{D1} + (1-\bar{D}-\tilde{d})(\bar{V}_{OUT} + \tilde{v}_{OUT}), \tag{9}$$

$$\frac{(\bar{V}_{OUT} + \tilde{v}_{OUT})}{R_{OUT}} = (1-\bar{D}-\tilde{d})(\bar{I}_{L1} + \tilde{i}_{L1}) - C_{OUT}\frac{d}{dt}(\bar{V}_{COUT} + \tilde{v}_{COUT}), \tag{10}$$

$$\bar{V}_{OUT} + \tilde{v}_{OUT} = \bar{V}_{COUT} + \tilde{v}_{COUT} + R_{COUT}C_{OUT}\frac{d}{dt}(\bar{V}_{COUT} + \tilde{v}_{COUT}) \tag{11}$$

Third, by applying the linearization technique (assume that the high order non-linear terms are small and negligible), a set of DC and AC equations can be obtained as follows:

DC equations:

From (9)

$$\bar{V}_{IN} = (1-\bar{D})V_{D1} + (1-\bar{D})\bar{V}_{OUT}$$

$$\bar{D} = \frac{\bar{V}_{OUT} - \bar{V}_{IN} + V_{D1}}{\bar{V}_{OUT} + V_{D1}}. \tag{12}$$

Also

$$\bar{V}_{OUT} = \frac{\bar{V}_{IN} - (1-\bar{D})V_{D1}}{(1-\bar{D})}.$$

For simplicity, consider that $(1-\bar{D})V_{D1}$ is small compared with $\bar{V}_{IN}$,

$$\bar{V}_{OUT} = \frac{\bar{V}_{IN}}{(1-\bar{D})}. \tag{13}$$

From (10),

$$\bar{I}_{L1} = \frac{\bar{V}_{OUT}}{(1-\bar{D})R_{OUT}}. \quad (14)$$

AC equations:

From (9),

$$\tilde{v}_{IN} = sL_1\tilde{i}_{L1} + (1-\bar{D})\tilde{v}_{OUT} - \tilde{d}\bar{V}_{OUT}$$

$$sL_1\tilde{i}_{L1} = \tilde{v}_{IN} - (1-\bar{D})\tilde{v}_{OUT} + \tilde{d}\bar{V}_{OUT} \quad (15)$$

From (10),

$$\frac{\tilde{v}_{OUT}}{R_{OUT}} = (1-\bar{D})\tilde{i}_{L1} - \tilde{d}\bar{I}_{L1} - sC_{OUT}\tilde{v}_{COUT}, \quad (16)$$

From (11),

$$\tilde{v}_{OUT} = \tilde{v}_{COUT} + sR_{COUT}C_{OUT}\tilde{v}_{COUT},$$

$$\tilde{v}_{COUT} = \frac{\tilde{v}_{OUT}}{1 + sR_{COUT}C_{OUT}}, \quad (17)$$

Substitute (13), (14), (15), and (17) into (16),

$$\tilde{v}_{OUT} = \frac{\dfrac{(1+sR_{COUT}C_{OUT})\tilde{v}_{IN}}{(1-\bar{D})} + \dfrac{(1+sR_{COUT}C_{OUT})\bar{V}_{IN}}{(1-\bar{D})^2}\left[1 - \dfrac{sL_1}{R_{OUT}(1-\bar{D})^2}\right]\tilde{d}}{1 + s\left[\dfrac{L_1}{R_{OUT}(1-\bar{D})^2} + R_{COUT}C_{OUT}\right] + s^2\dfrac{L_1C_{OUT}(R_{OUT}+R_{COUT})}{R_{OUT}(1-\bar{D})^2}} \quad (18)$$

Modeling of a Current Mode Control Boost Converter

Under current mode control, i_{L1} is fed back when Q_1 is turned on to determine the on-time of Q_1 by comparing it to a current control signal i_C. A compensation ramp of slope $-m_C$ is normally added to avoid sub-harmonic oscillation. Figure 2 shows the current waveform of an on period. The averaged state equation is as follows:

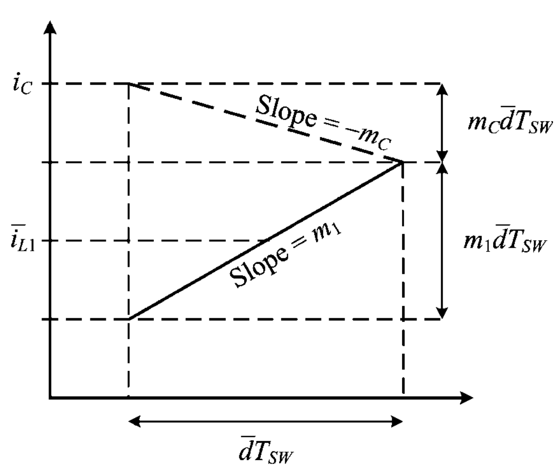


FIGURE 2. Inductor Current Waveform at the On Period

$$\bar{i}_{L1} = i_C - m_C\bar{d}T_{SW} - \frac{m_1}{2}\bar{d}T_{SW}, \quad (19)$$

where

$$m_1 = \frac{v_{IN}}{L_1} \quad (20)$$

is the slope of i_{L1} during the on period. By adding small signal perturbations to the above equation,

$$\bar{I}_{L1} + \tilde{i}_{L1} = \bar{I}_C + \tilde{i}_C - m_C(\bar{D}+\tilde{d})T_{SW} - \frac{T_{SW}}{2}(\bar{M}_1 + \tilde{m}_1)(\bar{D}+\tilde{d}). \quad (21)$$

By applying the linearization technique (assume that the high order non-linear terms are small and negligible), a set of DC and AC equations can be obtained as follows:

DC equation:

$$\bar{I}_{L1} = \bar{I}_C - m_C\bar{D}T_{SW} - \frac{T_{SW}}{2}\bar{M}_1\bar{D} \quad (22)$$

where

$$\bar{M}_1 = \frac{\bar{V}_{IN}}{L_1}$$

AC eqaution:

$$\tilde{i}_{L1} = \tilde{i}_C - m_CT_{SW}\tilde{d} - \frac{T_{SW}}{2}\bar{M}_1\tilde{d} - \frac{T_{SW}}{2}\bar{D}\tilde{m}_1 \quad (23)$$

Define

$$T_2 = \frac{T_{SW}}{2}, \quad (24)$$

$$T_M = \frac{T_{SW}}{2}(2m_C + \bar{M}_1) \quad (25)$$

From (20), (23), (24), (25),

$$\tilde{i}_{L1} = \tilde{i}_C - T_M\tilde{d} - \bar{D}T_2\frac{\tilde{v}_{IN}}{L_1}. \quad (26)$$

Substitute (13) into (15),

$$sL_1\tilde{i}_{L1} = \tilde{v}_{IN} - (1-\bar{D})\tilde{v}_{OUT} + \tilde{d}\frac{\bar{V}_{IN}}{(1-\bar{D})}. \quad (27)$$

Substitute (27) into (26),

$$\tilde{d} = \frac{sL_1\tilde{i}_C + (1-\bar{D})\tilde{v}_{OUT} - \left(1 + \dfrac{sL_1\bar{D}T_2}{L_1}\right)\tilde{v}_{IN}}{\left[\dfrac{\bar{V}_{IN}}{(1-\bar{D})} + sL_1T_M\right]}. \quad (28)$$

By substituting (28) into (18), the small signal model of the current mode control boost converter can be formulated as follows:

$$\tilde{v}_{OUT} = \frac{G_{NC}(s)\tilde{v}_{IN} + G_{IC}(s)\tilde{i}_C}{\Delta(s)}, \quad (29)$$

where

$$G_{NC}(s) = R_{OUT}(1 - \bar{D})(1 + sR_{COUT}C_{OUT})\left\{\frac{(1 - \bar{D})T_M}{V_{IN}} + \frac{1}{R_{OUT}(1 - \bar{D})^2} - \frac{\bar{D}T_2}{L_1} + \frac{s\bar{D}T_2}{R_{OUT}(1 - \bar{D})^2}\right\}$$

$$G_{IC}(s) = R_{OUT}(1 - \bar{D})(1 + sR_{COUT}C_{OUT})\left[1 - s\frac{L_1}{R_{OUT}(1 - \bar{D})^2}\right]$$

$$\Delta(s) = 2 + R_{OUT}(1 - \bar{D})^2 T_M \frac{(1 - \bar{D})}{V_{IN}} + s\left\{\left[L_1 + R_{COUT}R_{OUT}C_{OUT}(1 - D)^2\right]T_M\frac{(1 - \bar{D})}{V_{IN}} + (R_{OUT} + 2R_{COUT})C_{OUT}\right\} + s^2\left[L_1C_{OUT}(R_{OUT} + R_{COUT})T_M\frac{(1 - \bar{D})}{V_{IN}}\right]$$

If the current i_{L1} is sensed by a resistor R_{SN} connecting between Q_1 and the ground, the current control signal i_C can be converted to a voltage control signal v_C. The relationship between the output voltage and the voltage control signal can be formulated as follows:

$$\tilde{v}_{OUT} = \frac{G_{IC}(s)}{\Delta(s)R_{SN}}\tilde{v}_C, \qquad (30)$$

Principle Of a Lag Compensator

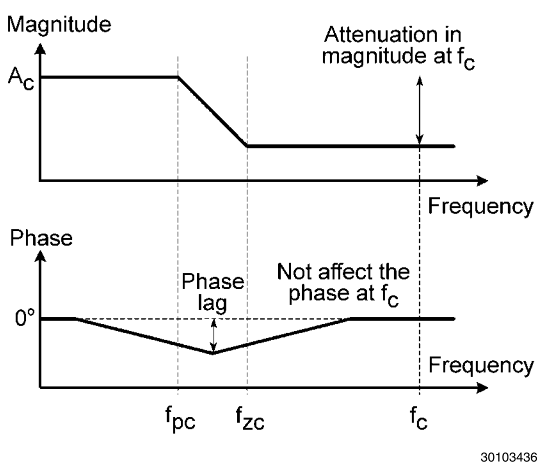


FIGURE 3. Frequency Response of a Lag Compensator

A lag compensator consists of a pair of pole and zero at the frequency f_{PC} and f_{ZC}, with $f_{PC} < f_{ZC}$. It also provides a dc gain A_C. As shown in Figure 3, the lag compensator provides an attenuation in magnitude at the high frequency. The degree of attenuation is determined by the distance between f_{PC} and f_{ZC}. It is because the magnitude is decreased at a slope of 20dB/decade between f_{PC} and f_{ZC}. The lag compensator also provides a phase lag. However, f_{PC} and f_{ZC} can be placed at a low frequency (much lower than the frequency of interest, e.g. the cross over frequency f_C) such that the lag compensator nearly does not affect the phase at the high frequency.

The aim of designing a lag compensator is to provide a desired phase margin for the compensated system. Starting from a bode plot of an un-compensated system, and a requirement of phase margin of Φ_m, a new f_C can be selected at the frequency corresponding to 180° - Φ_m of the un-compensated system. Then the magnitude of the un-compensated system at f_C is found. The magnitude at f_C can be attenuated to 0dB by the lag compensator through proper design of f_{PC} and f_{ZC}. As a result, the compensated system will have a phase margin of Φ_m, and the cross over frequency will be f_C. An illustrative example will be presented to show the design steps.

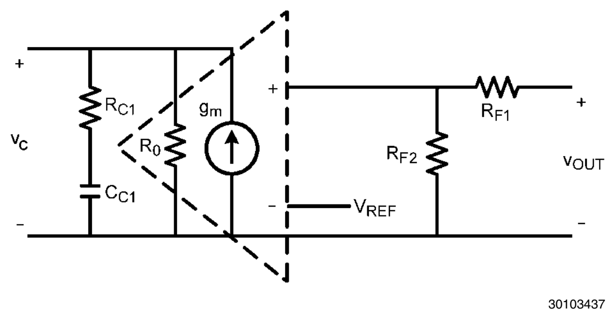


FIGURE 4. A Lag Compensator Implemented by a Transconductance Amplifier Circuit

A lag compensator can be implemented by a transconductance amplifier, with an open loop gain of gm and an output impedance of R_0, connecting to a resistor R_{C1} and a capacitor C_{C1} in series to the ground, as shown in Figure 4. Let the negative input of the amplifier is connected to a reference voltage V_{REF}, and the positive input is connected to the output voltage v_{OUT} through a resistor divider network implemented by R_{F1} and R_{F2}, the transfer function relating v_C and v_{OUT} is

$$v_C = \left(\frac{R_{F2}}{R_{F1} + R_{F2}}v_{OUT} - V_{REF}\right)g_m\left[R_0 // \left(R_{C1} + \frac{1}{SC_{C1}}\right)\right] \qquad (31)$$

By adding small signal perturbations, the AC equation can be obtained as follows:

$$\tilde{v}_C = \frac{R_{F2}}{R_{F1} + R_{F2}}g_mR_0\frac{1 + sR_{C1}C_{C1}}{1 + s(R_{C1} + R_0)C_{C1}}\tilde{v}_{OUT}. \qquad (32)$$

Hence,

$$A_C = \frac{R_{F2}}{R_{F1} + R_{F2}}g_mR_0,$$

$$f_{PC} = \frac{1}{2\pi(R_{C1} + R_0)C_{C1}},$$

$$f_{ZC} = \frac{1}{2\pi R_{C1}C_{C1}}.$$

Illustrative Example

The design of a current mode control Boost converter with a nominal input voltage of 5V and an output voltage of 12V and an output current of 0.5A will be shown. The major components are listed in table 1. A current mode controller LM3478 will be used. The parameters of the LM3478, which can be derived from the datasheet, are also listed in table 2.

TABLE 1. Major Parameters of the Example Boost Converter

Parameter	Value
$\overline{V}_{IN}$	5V
$\overline{V}_{OUT}$	12V
R_{OUT}	24Ω
L_1	10 µH
C_{OUT}	150 µF
R_{COUT}	0.05Ω
f_{SW}	400 kHz
R_{SN}	0.05Ω
R_{SL}	604Ω

TABLE 2. Parameters of the LM3478

Parameter	Value
V_{REF}	1.26V
g_m	800 $\mu\Omega^{-1}$
R_0	$A_V/g_m = 38/800\ \mu\Omega^{-1}$ = 47.5 kΩ
V_{SL}	92 mV

Other parameters of (30) are calculated below.

From (13),

$$\overline{D} = 0.5833$$

From (24),

$$T_2 = \frac{T_{SW}}{2} = \frac{1}{2f_{SW}}$$

$$T_2 = 1.25\ \mu s$$

The parameter m_C is determined by an internal compensation ramp V_{SL} and an external compensation ramp determined by an internal current of 40 µA passing through an external resistor R_{SL}. It can be calculated by the following equation:

$$m_C = (V_{SL} + 40\ \mu A \times R_{SL})f_{SW}/R_{SN} = 929280As^{-1}$$

From (25),

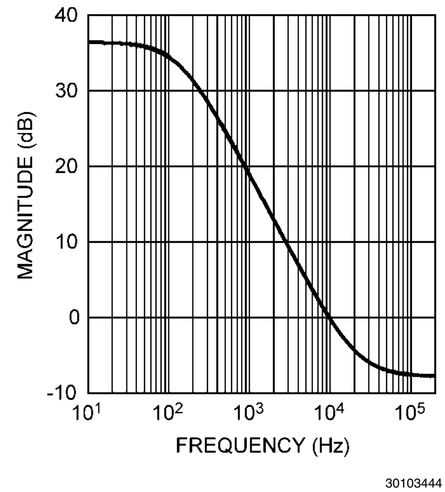
$$T_M = \frac{T_{SW}}{2}\left(2m_C + \frac{\overline{V}_{IN}}{L_1}\right)$$
$$= 2.9482A$$

Hence, all parameters of (30) are obtained. A bode plot of (30) with the above parameters is shown in Figure 5. The DC gain, zeros, and poles are

DC gain: 36.39dB

Zeros: 53 kHz, 66 kHz (right half plane zero)

Poles: 133 Hz, 65 kHz



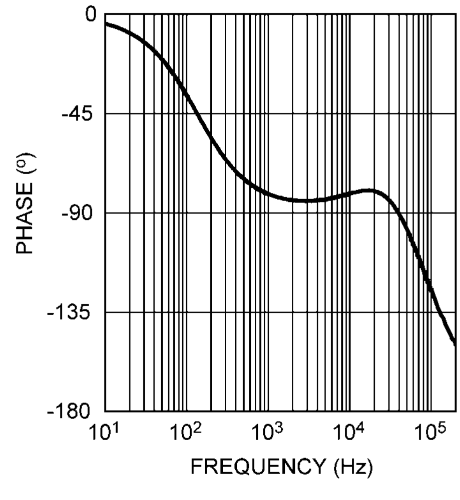


FIGURE 5. Frequency Response of the Un-Compensated System

The design of the lag compensator can be achieved by following (32). Since $\overline{V}_{OUT}$ and V_{REF} are 12V and 1.26V respectively, we can design that

$$R_{F1} = 84.5\ k\Omega$$

$$R_{F2} = 10\ k\Omega$$

From (32),

$$A_C = \frac{R_{F2}}{R_{F1} + R_{F2}} g_m R_0$$
$$= 4.02$$
$$= 12.09\ dB$$

In this example, a compensated system with a phase margin of around 90° is desired. From Figure 5, f_C can be selected as 3.5kHz (the frequency at which the phase is around 180° - 90° = 90°). The attenuation required is 7dB + A_C = 19.09dB, which implies that the distance between f_{PC} and f_{ZC} should be 0.96 decade. Select f_{ZC} to be 350 Hz, i.e. one decade lower than f_C, then f_{PC} should be 38.3 Hz.

$$1/R_{C1}C_{C1} = 2\pi \times 350\ Hz$$

$$\frac{1}{(R_{C1}+R_0)C_{C1}} = 2\pi \times 38.3\text{ Hz}$$

$$R_0C_{C1} = \frac{1}{2\pi \times 38.3\text{ Hz}} - R_1C_{C1}$$

$$C_{C1} = 78\text{ nF}$$

$$R_{C1} = 5.85\text{ k}\Omega$$

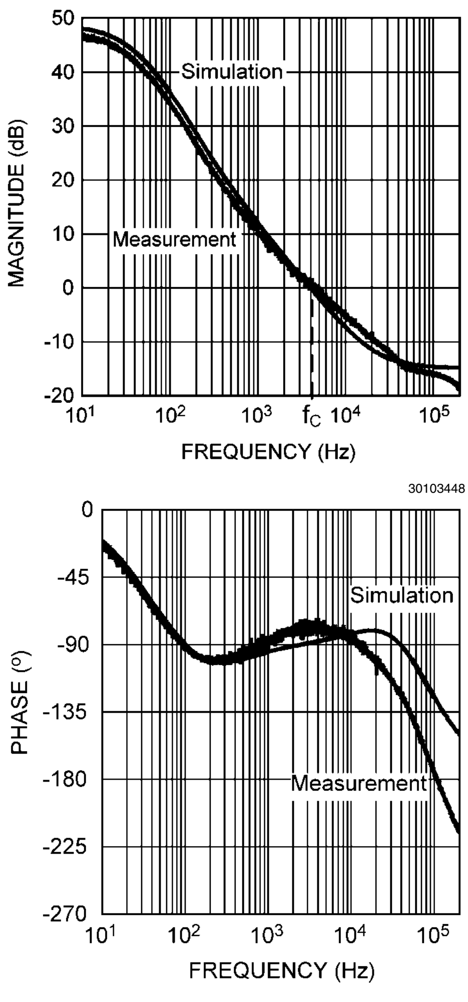


FIGURE 6. Frequency Response of the Compensated System with 90° Phase Margin

Finally, select R_{C1} = 5.9 kΩ and C_{C1} = 100 nF. The frequency response of the compensated system is shown in Figure 6. It can be found that the 0dB point is at around 4 kHz, and the phase margin is around 95°.

Conclusion

This application note details the modeling of an open loop and a current mode control boost converter under the continuous conduction mode. The principle and design of a lag compensator have also been addressed. An example has been presented to illustrate the design. A lag compensator has been designed for a compensated system with around 90° phase margin.

The selection of a desired phase margin affects the transient response of the output voltage. Moreover, some practical systems are suffered from noise, and transient responses are not a major concern. A lower cross over frequency may be required. In this case, the application of the dominant pole compensation method may be more appropriate. The lag compensator can be easily changed to a compensator with a dominant pole by setting R_{C1} to zero, i.e. to eliminate the zero. Application engineers are suggested to design properly based on practical situations.

Notes

For more National Semiconductor product information and proven design tools, visit the following Web sites at: www.national.com

Products		Design Support	
Amplifiers	www.national.com/amplifiers	WEBENCH® Tools	www.national.com/webench
Audio	www.national.com/audio	App Notes	www.national.com/appnotes
Clock and Timing	www.national.com/timing	Reference Designs	www.national.com/refdesigns
Data Converters	www.national.com/adc	Samples	www.national.com/samples
Interface	www.national.com/interface	Eval Boards	www.national.com/evalboards
LVDS	www.national.com/lvds	Packaging	www.national.com/packaging
Power Management	www.national.com/power	Green Compliance	www.national.com/quality/green
Switching Regulators	www.national.com/switchers	Distributors	www.national.com/contacts
LDOs	www.national.com/ldo	Quality and Reliability	www.national.com/quality
LED Lighting	www.national.com/led	Feedback/Support	www.national.com/feedback
Voltage References	www.national.com/vref	Design Made Easy	www.national.com/easy
PowerWise® Solutions	www.national.com/powerwise	Applications & Markets	www.national.com/solutions
Serial Digital Interface (SDI)	www.national.com/sdi	Mil/Aero	www.national.com/milaero
Temperature Sensors	www.national.com/tempsensors	SolarMagic™	www.national.com/solarmagic
PLL/VCO	www.national.com/wireless	PowerWise® Design University	www.national.com/training

For the most current product information visit us at www.national.com

National Semiconductor Americas Technical Support Center
Email: support@nsc.com
Tel: 1-800-272-9959

National Semiconductor Europe Technical Support Center
Email: europe.support@nsc.com

National Semiconductor Asia Pacific Technical Support Center
Email: ap.support@nsc.com

National Semiconductor Japan Technical Support Center
Email: jpn.feedback@nsc.com